SQUEEZED STATES AND GRAVITON-ENTROPY PRODUCTION IN THE EARLY UNIVERSE

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Abstract

Squeezed states are a very useful framework for the quantum treatment of tensor perturbations (i.e. gravitons production) in the early universe. In particular, the non equilibrium entropy growth in a cosmological process of pair production is completely determined by the associated squeezing parameter and is insensitive to the number of particles in the initial state. The total produced entropy may represent a significant fraction of the entropy stored today in the cosmic blackbody radiation, provided pair production originates from a change in the background metric at a curvature scale of the Planck order. Within the formalism of squeezed thermal states it is also possible to discuss the stimulated emission of gravitons from an initial thermal bath,under the action of the cosmic gravitational background field. We find that at low energy the graviton production is enhanced, if compared with spontaneous creation from the vacuum; as a consequence , the inflation scale must be lowered, in order not to exceed the observed CMB quadrupole anisotropy. This effect is important, in particular, for models based on a symmetry-breaking transition which require, as initial condition, a state of thermal equilibrium at temperatures higher than the inflation scale and in which inflation has a minimal duration.

1 Introduction

In order to discuss the graviton production induced by a cosmological background transition, the starting point is the linearized wave equation for a tensor perturbation. For the sake of generality we will take into account also the variation of Newton constant [1], which corresponds, in a string cosmological scenario [2], to a time-dependence of the Fradkin-Tseytlin dilaton field $\phi(t)$. The linearized wave equation is then, in the Brans-Dicke (Stringy) frame [1]

$$\Box h_i^j - \dot{\phi}\dot{h}_i^j = 0 \tag{1}$$

where h_i^j is the graviton field describing a tensor perturbation on a given curved background, represented by a homogeneous diagonal metric in which d dimensions expand with the scale factor $a(t)$ and n dimension contract with the scale factor $b(t)$:

$$g_{\mu\nu} \equiv diag\left(1, -a^2(t)\gamma_{ij}(x), -b^2(t)\gamma_{ab}(y)\right) \tag{2}$$

(conventions: $\mu, \nu = 1, ..., D = d + n + 1$; i,j=1,...,d; a,b=1,...,n; t is the cosmic time coordinate, ad γ_{ij}, γ_{ab} are the metric tensors of two maximally symmetric euclidean manifolds, parametrized respectively by "internal" and "external" coordinates x^i and y^a).

In terms of the conformal time coordinate η, defined by $dt/d\eta = a$, equation (1) becomes [1]

$$\psi_i^{j''} + \left(k^2 - V(\eta)\right)\psi_i^j = 0 \tag{3}$$

where $\psi_i^j = h_i^j a^{\frac{d-1}{2}} b^{\frac{n}{2}} e^{-\frac{\phi}{2}}$ and V is

$$V(\eta) = \frac{(d-1)}{2}\frac{a''}{a} + \frac{n}{2}\frac{b''}{b} + \frac{1}{4}(d-1)(d-3)\left(\frac{a'}{a}\right)^2 + \frac{1}{4}n(n-2)\left(\frac{b'}{b}\right)^2 +$$

$$+ \frac{1}{4}\phi'^2 + \frac{1}{2}n(d-1)\frac{a'b'}{ab} - \frac{1}{2}(d-1)\frac{a'}{a}\phi' - \frac{n}{2}\frac{b'}{b}\phi' \tag{4}$$

This effective potential takes into account the contribution of the expanding dimensions ($a' \neq 0$), of the contacting dimensions ($b' \neq 0$) and of the possible variation in time of the gravitational coupling constant($\phi' \neq 0$). In the case of four expanding dimensions (without dilaton field and without contracting dimensions) we recover the standard result, a minimally coupled scalar field equation.

The quantum description of the amplification of scalar or tensor fluctuations, as discussed here (for other references see e.g. [11]), is based on the separation of the field into background solution and first order perturbations, and on the expansion of the solution to the perturbed wave equation into $| in \rangle$ and $| out \rangle$ modes. The complex coefficients of this expansion are interpreted in second quantization formalism as annihilation and creation operators for a particle $(b, b^\dagger)$ and the corresponding antiparticle $(\tilde{b}, \tilde{b}^\dagger)$. The relation between $| in \rangle$ and $| out \rangle$ mode solution can thus be expressed for each mode k as a Bogoliubov transformation between the $| in \rangle$ operators $(b, b^\dagger, \tilde{b}, \tilde{b}^\dagger)$ and the out ones $(a, a^\dagger, \tilde{a}^\dagger, \tilde{a})$ [3]

$$a_k = c_+(k)b_k + c_-^*(k)\tilde{b}_{-k}^\dagger \quad , \quad \tilde{a}_{-k}^\dagger = c_-(k)b_k + c_+^*(k)\tilde{b}_{-k}^\dagger \tag{5}$$

where $|c_+|^2 - |c_-|^2 = 1$. As noted by Grishchuk and Sidorov [3], by parametrizing the Bogoliubov coefficients $c_\pm$ in terms of the two real numbers $r \geq 0$ and θ,

$$c_+(k) = \cosh r(k) \quad , \quad c_-^*(k) = e^{2i\theta_k}\sinh r(k) \tag{6}$$

the relations (1) can be re-written as unitary transformations generated by the (momentum-conserving) two-mode squeezing operator Σ_k,

$$\Sigma_k = \exp(z_k^* b_k \tilde{b}_{-k} - z_k b_k^\dagger \tilde{b}_{-k}^\dagger) \quad , \quad z_k = r(k)e^{2i\theta_k} \tag{7}$$

(r is the so-called squeezing parameter) as

$$a_k = \Sigma_k b_k \Sigma_k^\dagger \tag{8}$$

(and related expressions for $\tilde{a}^\dagger, \tilde{a}, \tilde{a}$)

The mean number of produced gravitons is then, according to equation (8)

$$\overline{N}_k = \langle 0|a_k^\dagger a_k|0\rangle = |c_-(k)|^2 = \sinh^2 r_k \qquad (9)$$

From equation (5) is possible to compute the spectral energy density

$$\rho(\omega) = \omega(\frac{d\rho_g}{d\omega}) \simeq \omega^4 \overline{N}(\omega), \qquad (10)$$

ω is the proper frequency related to the comoving one k by $\omega = k/a(t)$ where a(t) is the scale of the expanding background metric.

We then insert the known expression of $c_-(\omega)$ in eq.(9) and measure $\rho(\omega)$, as usual, in units of critical energy density ρ_c, defining $\Omega(\omega) = \rho(\omega)/\rho_c$. We have, in four dimensions $(D = 4)$ expanding with scale factor $a(\eta) \simeq \eta^{-\alpha}$ in conformal time [4]

$$\Omega(\omega, t_0) \simeq GH_1{}^2 \Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{2-2\alpha} \qquad , \qquad \omega_2 < \omega < \omega_1 \qquad (11)$$

$$\Omega(\omega, t_0) \simeq GH_1{}^2 \Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{2-2\alpha}(\frac{\omega}{\omega_2})^{-2} \qquad , \qquad \omega_0 < \omega < \omega_2. \qquad (12)$$

$\Omega_\gamma(t_0) \simeq 10^{-4}$ is the fraction of the critical energy density present today in the form of radiation; $\alpha \geq 1$ is a coefficient parametrizing (in conformal time) the power-law behaviour of the scale factor; $H_1 \equiv H(t_1)$ is the curvature scale at the time t_1 marking the end of inflation and the beginning of the radiation-dominated era; $\omega_0 \simeq 10^{-18}$ Hz is the minimum amplified frequency crossing today the Hubble radius H_0^{-1}; $\omega_2 \simeq 10^2 \omega_0$ is the frequency corresponding to the matter radiation transition; ω_1, finally, is the maximum amplified frequency, related to the inflation scale by $\omega_1 \simeq 10^{11}(H_1/M_P)^{1/2}$ Hz (M_P is the Planck mass).

The computed spectra are constrained by the CMB anisotropy, by the pulsar timing data and by the closure density. The bounds on the variation of the spectral energy density becomes bounds on the variation of the squeezing parameter, which is given by [1],[3]

$$r(\omega) = |\delta|[25 - ln(\frac{\omega}{Hz}) + \frac{1}{2} ln(\frac{H_1}{M_P})] \qquad (13)$$

($|\delta|$ is a model-dependent number of order of unity).

2 Thermal modification of the graviton spectrum

In this picture the crucial assumption is that the initial state of the gravitons is precisely the vacuum. The vacuum, however, is not the most general initial state for a gravity wave or for a generic scalar perturbation[5]. We can mimic a generic initial state with a squeezed number state, or, better with a statistical mixture of two mode squeezed number states [6]. In particular, any inflationary model based on a temperature dependent phase transition require as initial condition a homogeneous thermal state. So, a particularly relevant case is that of a thermal mixture of number states. Such initial condition will modify the mean number of particles and the spectral energy density .

For the spectral energy density we obtain [6]

$$\Omega(\omega, t_0) \simeq GH_1{}^2\Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{2-2\alpha}\coth(\frac{\beta_0\omega}{2}) \quad , \quad \omega_2 < \omega < \omega_1 \tag{14}$$

$$\Omega(\omega, t_0) \simeq GH_1{}^2\Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{2-2\alpha}(\frac{\omega}{\omega_2})^{-2}\coth(\frac{\beta_0\omega}{2}) \quad , \quad \omega_0 < \omega < \omega_2. \tag{15}$$

Here $\beta_0{}^{-1} \equiv \beta^{-1}(t_0)$ is the proper temperature of the initial thermal state, adiabatically rescaled down to the present observation time t_0 [β_0 is defined in terms of the comoving temperature $\bar{\beta}$ as $\beta(t_0) = \bar{\beta}a(t_0)$]. The effect of the initial finite temperature is to enhance graviton production at low frequency with respect to the high frequency sector of the spectrum. This effect depends on the value of the initial temperature which, in the context of inflationary models based on thermal symmetry breaking, is greater than the inflation scale. However, the modification of the spectrum is relevant only if the inflationary period is not too long (see [6] for a detailed discussion).

3 Entropy production from the cosmological amplification of vacuum fluctuations

Unlike the particle spectrum, which depends on the initial state, the non equilibrium entropy growth, associated with the process of particle production [7], is not affected by the particular choice of the initial conditions.

It is possible indeed to introduce a coarse graining approach to non equilibrium entropy, valid for squeezed states, in which the loss of information associated to the reduced density matrix is represented by the increased dispersion in the superfluctuant operators $x, \tilde{x}$ whose variance is amplified with respect to their initial value [8], [9]. In terms of these operators a and $\tilde{a}$ have the following differential representation [10]

$$a_k = \frac{i}{2}e^{i\vartheta_k}\left[(\cosh r_k - \sinh r_k)(x - i\tilde{x}) + (\cosh r_k + \sinh r_k)(\partial_x - i\partial_{\tilde{x}})\right] \tag{16}$$

$$\tilde{a}_{-k} = \frac{i}{2}e^{i\vartheta_k}\left[(\cosh r_k - \sinh r_k)(x + i\tilde{x}) + (\cosh r_k + \sinh r_k)(\partial_x + i\partial_{\tilde{x}})\right] \tag{17}$$

(the relative phase has been chosen with respect to ϑ_k, in such a way to identify the x and $\tilde{x}$ operators with the superfluctuant ones) and the squeezed number wavefunctions (in the basis of the superfluctuant operators)becomes

$$\psi_{n_k,n_k}(x, \tilde{x}) = <x\tilde{x} \mid \Sigma_k \mid n_k n_{-k} >=<x\tilde{x} \mid \frac{(a_k^\dagger \tilde{a}_{-k}^\dagger)^n}{n!}\Sigma_k \mid 0> = \left(\frac{\sigma_k}{\pi}\right)^{\frac{1}{2}} L_n^0\left(\sigma_k(x^2 + \tilde{x}^2)\right) \times$$

$$\times e^{-\frac{\sigma_k}{2}(x^2+\tilde{x}^2)} = e^{in(\pi-2\vartheta_k)}\left(\frac{\sigma_k}{\pi}\right)^{\frac{1}{2}} e^{-\sigma_k(x^2+\tilde{x}^2)/2}\sum_{m=0}^n \frac{1}{m!(n-m)!}H_{2m}(\sqrt{\sigma_k}x)H_{2n-2m}(\sqrt{\sigma_k}\tilde{x}) \tag{18}$$

where L_n and H_n are the Laguerre and the Hermite polynomials, respectively and $\sigma_k = e^{-2r_k}$. It should be noted that, because of pair corrrelations, this wavefunction cannot be simply factorized

in terms of two decoupled squeezed oscillators in an excited state, which are known to provide the usual representation for the one mode squeezed number wavefunction.

It is interesting to point out, in passing, that the wave functions of a two mode squeezed state, as well as the transition probability between a generic two mode number state and a two mode squeezed number state, in the superfluctuant variables representation $(x\ ,\ \tilde{x})$ are the same as the corresponding quantities obtained in the context of the "squeezed" Landau levels problem for the electron in a uniform magnetic field [12]. In the Landau levels problem the two quantum numbers labelling the one particle wave functions are the energy of the electron and the component of the angular momentum perpendicular to the plane of the classical motion of the electron. Here the two quantum numbers in the many particle wavefunction are respectively the number of gravitons with four-momentum k (n_k) and with four-momentum -k (n_{-k}). Also to be mentioned is the fact that it is possible, within this formalism, to consider more general wavefunctions with $n_k \neq n_{-k}$

This is physically equivalent to consider, as initial condition for the gravitons, a state with non-zero number of particles and non zero four-momentum.

The entropy growth for a generic squeezed mixture of number states is the same as for the squeezed vacuum [10],

$$\Delta S_k = -Tr(\rho_{sk}\ln\rho_{sk}) + [Tr(\rho_{sk}\ln\rho_{sk})]_{r_k=0} = 2r_k \tag{19}$$

where ρ_{sk} is the reduced density matrix for the mode k, and the integrated entropy over all the graviton spectrum is [8], [9] (from eq. (19), (13)):

$$S_g \simeq |\delta| S_\gamma (\frac{H_1}{M_P})^{3/2} \tag{20}$$

where $S_\gamma \simeq [a(t)\ell T_\gamma(t)]^3 = const$ is the usual black-body entropy of the CMB radiation (in terms of the to-day parameters, $(a_0\ell T_{\gamma 0})^3 \sim (T_{\gamma 0}/H_0)^3 \sim (10^{29})^3)$.

Particle production from the vacuum is thus a process able to explain the observed cosmological level of entropy provided the curvature scale at the inflation radiation transition is of the order of the Planck one [9], [10].

In the standard de Sitter inflationary scenario the curvature scale is bounded from the CMB anisotropy observations and has to be $H_1 \leq 10^{-5} M_{PL}$. This observation would seem to rule out the mechanism discussed here as a possible explanation of the entropy of the universe. On the other hand such constraints are evaded if the de Sitter phase (or the radiation dominated phase) is preceeded by a phase of growing curvature, like in the "pre-big-bang" models [2] which arise naturally in duality-symmetric string cosmology, and in which the curvature scale can approach Planckian values $(H_1 \leq M_{PL})$ [1],[2].

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